Zadaci za vježbu - Trigonometrijske funkcije realnog argumenta

1. Na brojevnoj kružnici označi navedenu točku i obilježni vrijednost trigonometrijskih funkcija u toj točki:

a) $E\left(\frac{28\pi}{3}\right)$ b) $E\left(-\frac{19\pi}{2}\right)$ c) $E(5445^\circ)$

2. Bez uporabe kalkulatora izračunaj:

a) $\sin\left(-\frac{345\pi}{4}\right)$ b) $\operatorname{ctg}\left(\frac{217\pi}{12}\right)$ c) $\cos(-225^\circ)$
d) $\operatorname{tg}(-75^\circ)$ e) $\cos\left(-\frac{38\pi}{3}\right)$ f) $\sin(150^\circ)$
g) $\cos(105^\circ)$ h) $\operatorname{tg}\left(\frac{61\pi}{4}\right)$ i) $\operatorname{ctg}(1225^\circ)$

3. Ako je $\sin x = -\frac{11}{61}$, $x \in \left\langle 7\pi, \frac{15\pi}{2}\right\rangle$, te $\operatorname{ctg} y = \frac{24}{7}$, $y \in \left\langle 8\pi, \frac{17\pi}{2}\right\rangle$, koliko je:

a) $\sin(x-y)$ b) $\cos 2x$ c) $\cos(x+y)$
d) $\operatorname{ctg}(x+y)$ e) $\frac{2\sin x - 3\cos^2 y}{\operatorname{tg} y - \operatorname{ctg}^3 y}$ f) $\operatorname{tg} 2x + \operatorname{tg} 2y$

4. Ako je $\cos x = -\frac{5}{13}$, $x \in \left\langle 11\pi, \frac{23\pi}{2}\right\rangle$, te $\operatorname{tg} y = \frac{7}{24}$, $y \in \left\langle 10\pi, \frac{21\pi}{2}\right\rangle$, koliko je:

a) $\cos(x+y)$ b) $\operatorname{ctg} 2x$ c) $\sin(x-y)$
d) $\operatorname{tg}(x-y)$ e) $\frac{3\cos y + \sin^2 x}{\operatorname{ctg} x + \operatorname{tg}^3 y}$ f) $\operatorname{ctg} 2x + \operatorname{ctg} 2y$

5. Ako je $\alpha + \beta = \frac{\pi}{3}$ te $\operatorname{tg}\alpha = \frac{2}{5}$, odredi $\operatorname{tg}\beta$.

6. Ako je $\alpha + \beta = \frac{\pi}{4}$ te $\operatorname{tg}\beta = \frac{1}{7}$, odredi $\operatorname{tg}\alpha$.

7. Odredi parnost funkcije:

a) $f(x) = 3x - \cos 2x$ b) $f(x) = \sin^3 x - \operatorname{tg}^5 x$ c) $f(x) = 4x^7 + \operatorname{ctg} x$
d) $f(x) = \dfrac{\sin^2 x + \cos^3 x}{x^3 - \operatorname{ctg} x}$ e) $f(x) = \dfrac{2x^3 - \cos x}{\sin x}$ f) $f(x) = \dfrac{5x^2 - 4\cos x}{x + \sin x}$

8. Pojednostavni:

a) $(1 + \operatorname{tg}^2 x) \cdot \cos^2 x$ b) $(1 + \operatorname{ctg}^2 x) \cdot \sin^2 x$ c) $1 + \sin^2 x + \cos^2 x$
d) $\dfrac{\cos^2 x}{1 - \cos^2 x} \cdot \operatorname{tg}^2 x$ e) $\dfrac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x}$ f) $\dfrac{\sin^2 x}{\sin^2 x - 1} \cdot \operatorname{ctg}^2 x$

9. Dokaži:

a) $\operatorname{tg}^2 x - \sin^2 x = \operatorname{tg}^2 x \cdot \sin^2 x$ b) $\operatorname{ctg}^2 x - \cos^2 x = \operatorname{ctg}^2 x \cdot \cos^2 x$ c) $(\operatorname{tg}^2 x - \sin^2 x) \cdot \operatorname{ctg}^2 x = \sin^2 x$
d) $\dfrac{\cos x}{\sin x \cdot \operatorname{ctg} x} = 1$ e) $\dfrac{\sin x}{\cos x \cdot \operatorname{tg} x} = 1$ f) $\dfrac{\sin x + \operatorname{tg} x}{\operatorname{tg} x} = 1 + \cos x$

10. Nacrtaj graf funkcije:

a) $f(x) = 2\sin\left(2x - \frac{2\pi}{3}\right)$ b) $f(x) = -\frac{3}{2}\sin\left(2x + \frac{\pi}{4}\right)$ c) $f(x) = 2\cos\left(\frac{3\pi}{4} - 2x\right)$

11. Odredi funkciju sa slike